Business PreCalculus MATH 1643 Section 004, Spring 2013

Lesson 2: Polynomials

Definition 1. **Polynomial:** *A polynomial has the form*

$$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} \cdots a_2 x^2 + a_1 x + a_0,$$

where

1. *The highest exponent $n \geq 0$ is called the* ***degree***.
2. *The constants $a_n, a_{n-1}, \cdots, a_1, a_0$ are called the* ***coefficients*** *of the polynomial.*
3. *a_n is called the* ***leading coefficient***.
4. *The monomials $a_n x^n, a_{n-1}x^{n-1}, \cdots, a_1 x, a_0$ are called the* ***terms*** *of the polynomial.*
5. *$a_n x^n$ is called the* ***leading term***.
6. *a_0 is called the* ***constant term***.

Note 1. 1. *The only polynomial that has no degree is the* ***Zero Polynomial***.

2. *The constant polynomial has degree* 0.

Definition 2. **Standard form:** *When the exponents of a polynomial decrease from left to right, then the polynomial is said to be in standard form.*

Example 1.

$$9 - 8m^3 + 4m + m^7,$$

is a polynomial whose standard form is

$$m^7 - 8m^3 + 4m + 9.$$

The degree of this polynomial is 7*, the leading coefficient is* 1*, the constant term is* 9*, and the leading term is m^7.*

Definition 3. **Adding and Subtracting Polynomials:** *To add polynomials: Group like terms and then combine them.*
To subtract two polynomials, first change the sign of each term in the second then group like terms and finally combine like terms.

Example 2. *Find the sum of the polynomials:*
$-4x^3 + 5x^2 + 7x - 2$ *and* $6x^3 - 2x^2 - 8x - 5$.
Solution:

$$\begin{aligned}
&(-4x^3 + 5x^2 + 7x - 2) + (6x^3 - 2x^2 - 8x - 5) \\
&= (-4x^3 + 6x^3) + (5x^2 - 2x^2) + (7x - 8x) + (-2 - 5) \\
&= 2x^3 + 3x^2 - x - 7.
\end{aligned}$$

Example 3. *Find the difference of the polynomials:*
$-4x^3 + 5x^2 + 7x - 2$ *and* $6x^3 - 2x^2 - 8x - 5.$
<u>Solution:</u>

$$
\begin{aligned}
&(-4x^3 + 5x^2 + 7x - 2) - (6x^3 - 2x^2 - 8x - 5) \\
&= (-4x^3 + 5x^2 + 7x - 2) + (-6x^3 + 2x^2 + 8x + 5) \quad \textit{(Change signs of each term in the second polynomial)} \\
&= (-4x^3 - 6x^3) + (5x^2 + 2x^2) + (7x + 8x) + (-2 + 5) \\
&= -10x^3 + 7x^2 + 15x + 3.
\end{aligned}
$$

Definition 4. **<u>Multiplying Polynomials:</u>** *To multiply two polynomials, multiply each term of one polynomial by every term of the other polynomial and combine like terms.*

Example 4. *Multiply:*
$4x^2 + 3x$ *and* $x^2 + 2x - 3.$
<u>Solution:</u>

$$
\begin{aligned}
&(4x^2 - 3x)(x^2 + 2x - 3) \\
&= 4x^2(x^2 + 2x - 3) - 3x(x^2 + 2x - 3) \\
&= 4x^2(x^2) + 4x^2(2x) + 4x^2(-3) - 3x(x^2) - 3x(2x) - 3x(-3) \\
&= 4x^4 + 8x^3 - 12x^2 - 3x^3 - 6x^2 + 9x \\
&= 4x^4 + (8x^3 - 3x^3) + (-12x^2 - 6x^2) + 9x \\
&= 4x^4 + 5x^3 - 18x^2 + 9x.
\end{aligned}
$$

Definition 5. **<u>FOIL Method for (A+B)(C+D):</u>**

$$(A + B)(C + D) = A \overset{F}{\cdot} C + A \overset{O}{\cdot} D + B \overset{I}{\cdot} C + B \overset{L}{\cdot} D$$

Example 5. *Multiply:*
$x + 2$ *and* $x - 7.$
<u>Solution:</u>

$$
\begin{aligned}
&(x + 2)(x - 7) \\
&= x.x + x.(-7) + 2.x + 2.(-7) \\
&= x^2 - 7x + 2x - 14 \\
&= x^2 - 5x - 14.
\end{aligned}
$$

Definition 6. **<u>Important Products to Remember:</u>**

1. $A^2 - B^2 = (A + B)(A - B)$ *Difference of squares*

2. $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$ *Difference of cubes*

3. $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ *Sum of cubes*

4. $A^2 - 2AB + B^2 = (A - B)^2$ *Perfect square, negative middle term*

5. $A^2 + 2AB + B^2 = (A + B)^2$ *Perfect square, positive middle term*